# QUESTIONS AND DISCUSSIONS.

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## NEW QUESTION.

**34.** Given the mixed integral and functional equation

$$\int_{x=0}^{x=x} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{x}{2}\right) + f(x)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

## DISCUSSIONS.

### I. CONCERNING AN ILLUSTRATION OF A CERTAIN NECESSARY CONDITION IN MINIMIZING A DEFINITE INTEGRAL WITH DISCONTINUOUS INTEGRAND.

By PAUL R. RIDER, Washington University.

The problem of minimizing a definite integral in which the integrand possesses a finite discontinuity along a plane curve has been considered by Bliss and Mason.[1] A certain necessary condition that they have discovered may be stated as follows:

*If a curve C, of parameter t, which passes from the fixed point $P_0$ to the fixed point $P_2$ and crosses a curve D, whose equations are $x = x(\alpha)$, $y = y(\alpha)$, minimizes the sum of the two integrals*

$$I = \int_{t_0}^{t_1} F(x, y, x', y')dt, \qquad i = \int_{t_1}^{t_2} f(x, y, x', y')dt,$$

*the first integral to be taken from the point $P_0$ to the curve D and the second from the curve D to the point $P_2$, then at $P_1$, the point of intersection of C and D, the relation*

$$x_a(F_{x'} - f_{x'}) + y_a(F_{y'} - f_{y'}) = 0$$

*must hold. The argument of $x_a$, $y_a$ is the value of $\alpha$ for the point $P_1$; the arguments of $F_{x'}$, $F_{y'}$ are the values of $x$, $y$, $x'$, $y'$ on the curve $C_{01}$ at the point $P_1$, and those of $f_{x'}$, $f_{y'}$ are the values of the same variables on the curve $C_{12}$ at the point $P_1$.*

This condition is well illustrated in a problem (Calculus 389) which recently appeared in the MONTHLY:

> A man is at the southeast corner of a section of land and wishes to walk to the opposite corner in the least possible time. A circular track with a radius of $1/\pi$ miles is located in the section tangent to the west line at a point 120 rods from the south line. Conditions are such that he can walk at the rate of 4 miles an hour inside the track and 3 miles an hour outside the track. What course should he choose and how long is it? See the figure.

Solutions of the problem were given in the MONTHLY, Vol. 23, No. 24 (April, 1916), p. 125 by H. S. Uhler, and p. 127 by A. H. Holmes.

[1] BLISS and MASON, "A Problem of the Calculus of Variations in which the Integrand is Discontinuous," *Transactions of the American Mathematical Society*, Vol. 7 (1906), pp. 325–336.